AdS/CFT boundary conditions, multi-trace perturbations, and the c-theorem.

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Abstract

We discuss possible choices for boundary conditions in the AdS/CFT correspondence, and calculate the renormalisation group flow induced by a double-trace perturbation. In running from the UV to the IR there is a unit shift in the central charge. The discrepancy between our result and results obtained by other authors is accounted for by the discovery that there is a non-trivial flow for perturbations induced by bulk fields with masses saturating the Breitenlohner-Freedman bound.

1 Introduction

The AdS/CFT correspondence [1], in relating conventional quantum gauge field theories to gravitational and string theories in higher dimensions, has proved to be of great importance in elucidating non-perturbative aspects of both kinds of theory, and is likely to retain a central role for some time to come. Most of the work that has been done on this correspondence has involved taking the large-N limit of the gauge theory, which corresponds to the classical limit of the gravity theory. But it is of considerable interest to go beyond this limit and consider loop corrections on the gravity side, which give $O(1/N)$ corrections to the gauge theory.

The calculation of string loops in AdS backgrounds is difficult, because the cancellation of divergences is not well understood, and the Ramond-Ramond fields make calculations in the string genus expansion largely intractable. However, for the calculation of certain quantities it is sufficient to consider loop corrections in the Supergravity limit of string theory. This is particularly true when there are non-renormalisation theorems protecting the quantities on the gauge theory side.

It is also interesting to consider relevant perturbations of the gauge theory that correspond to tachyonic fields in the Supergravity theory. These break the conformal symmetry of the boundary gauge theory, and drive a renormalisation group flow. To understand the effects of these perturbations on the gauge theory requires an understanding of their asymptotic behaviour near the boundary, and the boundary conditions that we can impose on them. These boundary conditions were first considered in [8, 13], though the older work of [2] is also relevant. Boundary condtions were also discussed in [12]. For tachyonic modes whose masses lie in an appropriate range, the difference between the ultraviolet and infrared fixed points is just a difference in boundary conditions. We will describe these boundary conditions from a Hamiltonian perspective in which this difference corresponds to the choice of Dirichlet or Neumann boundary conditions for the bulk field. The partition functions of the perturbed boundary theory are then related by a functional Fourier transform (in the large-N limit a saddle point approximation reduces this to a Legendre transform).

In a series of publications we have obtained results on one-loop Weyl anomalies in AdS/CFT [4, 5, 6] that reproduce the exact form of the anomaly on the gauge theory side, including 1/N corrections. The coefficients of these anomalies are central charges that, according to the holographic c-theorem, should be larger for the infrared fixed point than the ultraviolet one, for the range of scaling dimensions where both correspond to normalisable perturbations. The main purpose of this paper is to verify this using our calculation of the Weyl anomaly.

The result we obtain is in contradiction with the work of [7], which was made use of in other studies of the renormalisation group flow in AdS and dS spaces [18]. The calculation of [7] assumed that there is no flow for bulk masses saturating the Breitenlohner-Freedman bound. We will show that this is not true; there is a non-trivial flow in this case that can be related to an ambiguity in the boundary term [9, 10]. Hopefully this accounts for the discrepancy with our result for the anomaly, which is linear in the scaling dimension of the field.

This paper is organised as follows: in Section 2 we discuss boundary conditions for

AdS fields. In Section 3 we discuss canonical quantisation. In Section 4 we discuss the AdS/CFT correspondence formulae for single and multi-trace perturbations, and show how they are related to boundary conditions for bulk fields. In Section 5 we review our calculation of the Weyl anomaly and calculate the running of the central charge for a double-trace perturbation.

2 Boundary Conditions on AdS Fields

We write the metric of Euclidean AdS_{d+1} as

$$ds^2 = \frac{1}{z^2}\left(dz^2 + \sum_{i=1}^{d} dx_i^2\right) = dr^2 + z^{-2}\sum_{i=1}^{d} dx_i^2, \tag{1}$$

where $z = \exp(r)$, and we have set the length scale of AdS to unity. The boundary is at $z = 0$. Now consider a scalar field of mass m propagating in this metric. Near the boundary it has the asymptotic behaviour

$$\phi = \alpha(x)z^{d-\Delta} + \beta(x)z^{\Delta} + \ldots, \tag{2}$$

where Δ is a root of the equation

$$\Delta(\Delta - d) = m^2. \tag{3}$$

For $\Delta = d/2$ the second asymptotic solution goes like $z^{d/2}\ln z$. From a Hamiltonian point of view α and β are conjugate variables, in a sense that we will make more precise shortly.

Let us consider more carefully how the asymptotic form (2) of bulk fields near the boundary is related to the boundary condition. The action for a free scalar field in AdS is given by

$$\int d^{d+1}x\sqrt{g}\frac{1}{2}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right), \tag{4}$$

with linear variation

$$\int d^{d+1}x\sqrt{g}\delta\phi\left(-\nabla^2 + m^2\right)\phi + \frac{1}{2}\int d^dx z^{-d+1}\left(\phi\partial_z\delta\phi - (\partial_z\phi)\delta\phi\right). \tag{5}$$

For this variation to vanish, we need ϕ to obey the classical equations of motion in the bulk, with the boundary condition [13]

$$\partial_r\phi|_{\partial AdS} = \omega\phi|_{\partial AdS}, \tag{6}$$

where ω is arbitrary. If we take $\omega = \Delta$ and insert the asymptotic form (2) for ϕ near the boundary, this becomes

$$z^{\Delta-d}(\partial_r - \Delta)\phi|_{\partial AdS} = 0, \tag{7}$$

where we multiplied by $z^{d-\Delta}$ to obtain a relation that is finite at $z=0$. This condition diagonalises the value of α, but places no restriction on β. So we can identify α, β as conjugate variables.

Suppose that Δ is the smaller root of (3). If we make the change of variables $\tilde{\phi} = z^{-\Delta}\phi$ (so that $\tilde{\phi} \sim \beta$ at the boundary) then we see that (7) corresponds to Neumann boundary conditions for $\tilde{\phi}$. If Δ is the larger root of (3) then we can change variables to $\bar{\phi} = z^{\Delta-d}\phi$ so that $\bar{\phi} \sim \alpha$ at the boundary, and then (7) corresponds to Dirichlet boundary conditions for $\bar{\phi}$. Henceforth we will use Δ to denote the larger root of (3).

In the case where $\Delta = d/2$, we can change variables to $z^{-d/2}\phi$ and impose either Dirichlet or Neumann boundary conditions on this field, corresponding again to diagonalising the two possible asymptotics of the field.

At the quantum level, the above boundary conditions can be imposed by adding a boundary term to the action; these boundary terms may be renormalised by interactions of the bulk field.

3 Canonical Quantisation

Consider a free scalar field of mass m. For greater generality it is convenient to perturb the AdS_{d+1} metric in such a way that the boundary has a d-dimensional Einstein metric $\hat{g}$ [5]. The perturbed metric is

$$ds^2 = G_{\mu\nu}\, dX^\mu\, dX^\nu = dr^2 + z^{-2}\, e^\rho \hat{g}_{ij}(x)\, dx^i dx^j\,, \quad e^{\rho/2} = 1 - C\, z^2\,, \quad C = \frac{l^2 \hat{R}}{4\, d(d-1)}\,, \tag{8}$$

where $\hat{R}$ is the Ricci tensor on the boundary, and the $d+1$ dimensional Einstein equations are still satisfied. The action for the scalar field in this metric can be written as

$$\begin{aligned} S_\phi &= \frac{1}{2}\int d^{d+1}X\sqrt{G}\left(G^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + m^2\phi^2\right) \\ &= \frac{1}{2}\int \frac{d^dx\, dr}{z^4}\sqrt{\hat{g}}\, e^{2\rho}\,\left(\dot{\phi}^2 + z^2 e^{-\rho}\hat{g}^{ij}\partial_i\phi\,\partial_j\phi + m^2\phi^2\right)\,, \end{aligned} \tag{9}$$

where the dot denotes differentiation with respect to r. The norm on fluctuations of the field, from which the functional integral volume element $\mathcal{D}\phi$ can be constructed is

$$||\delta\phi||^2 = \int d^{d+1}X\sqrt{G}\,\delta\phi^2 = \int \frac{d^dx\, dr}{z^d}\sqrt{\hat{g}}\, e^{\frac{d}{2}\rho}\,\delta\phi^2\,. \tag{10}$$

We will interpret the co-ordinate r as Euclidean time, so to write down a Schrödinger equation we first re-define the field by setting $\phi = z^{\frac{d}{2}}\, e^{-\frac{d}{4}\rho}\varphi$ to make the 'kinetic' term in the action into the standard form, and remove the explicit r-dependence from the integrand of the norm. The action becomes

$$S_\phi = \frac{1}{2}\int d^dx\, dr\sqrt{\hat{g}}\,\left(\dot{\varphi}^2 + z^2 e^{-\rho}\varphi\left(\Box + \frac{(d-2)\hat{R}}{4(d-1)}\right)\varphi + \left(m^2 + \frac{d^2}{4}\right)\varphi^2\right)$$

$$
\begin{aligned}
& -\frac{1}{2}\int d^dx\,\sqrt{\hat g}\left(\frac{d}{4}\dot\rho+\frac{d}{2}\right)\varphi^2\,, \\
= \;& S_\varphi + S_b\,, \qquad (11)
\end{aligned}
$$

where $\Box$ is the d-dimensional covariant Laplacian constructed from $\hat g$. Note that $\Box+\frac{(d-2)\hat R}{4(d-1)}$ is the operator associated with a conformally coupled d-dimensional field, and the mass has been modified to an effective mass $M_r=\sqrt{m^2+d^2/4}$.

In the first instance we will consider diagonalising the boundary value of φ, so that $\dot\varphi$ is represented by functional differentiation acting on a boundary wave-functional. This wave-functional can be represented by a path integral

$$
\int \mathcal{D}\phi\, e^{-S_\phi}\Big|_{\phi(r=r_0)=\hat\phi} = e^{-S_b}\int \mathcal{D}\varphi\, e^{-S_\varphi}\Big|_{\varphi(r=r_0)=\hat\varphi} \equiv e^{-S_b+W[\hat\varphi,g]}, \qquad W[\hat\varphi]=F+\tfrac{1}{2}\int d^dx\sqrt{\hat g}\hat\varphi\Gamma\hat\varphi, \qquad (12)
$$

where a regularisation is achived by taking the boundary at $r=r_0$ rather than $r=-\infty$.

This satisfies a functional Schrödinger equation that can be read off from the action and gives

$$
\frac{\partial}{\partial r_0}\Gamma = \Gamma^2-\tau^2e^{-\rho}\left(\Box+\frac{(d-2)\hat R}{4(d-1)}\right)+M_r^2, \qquad \frac{\partial}{\partial r_0}F=\tfrac{1}{2}\mathrm{Tr}\Gamma, \qquad (13)
$$

solved by expanding Γ in powers of the differential operator [5]. The result is easily summed in terms of Bessel functions. To get the correct scaling dimension as $r_0\to-\infty$ requires discarding terms of order less than τ^{2M_r} in the asymptotic expansion of Γ. Such terms should be removed by an appropriate renormalisation. Namely we discard these terms so that Γ has the asymptotic behaviour

$$
\Gamma \sim \tau^{2M_r}p^{2M_r}. \qquad (14)
$$

Here $\tau=\ln r_0$ is the boundary value of z. To get a finite wave-functional as the cutoff is removed, we perform a wave-function renormalisation $\hat\varphi\to\tau^{-M_r}\hat\varphi$. The scaling dimension of our canonical field can be read off the asymptotic form of the wave-functional, and has the correct value (by construction). Now $\hat\varphi$ corresponds to α in (2), where the scaling dimension Δ is taken to be the larger root of (3). This is in accordance with our previous discovery that diagonalising α corresponds to a Dirichlet condition on a suitably defined bulk field.

To get the other condition, we can perform a functional Fourier transform on the boundary. This gives us a Neumann condition for our "canonical" field, so that the wave-functional is written in terms of a boundary value π for the field $\dot\varphi$. When the wave-function renormalisation is taken into account, it is clear that π must undergo a wave-function renormalisation $\pi\to\tau^{M_r}\pi$. Then π corresponds to the other asymptotic in (2) and has the correct scaling dimension. Henceforth when we talk about imposing Dirichlet or Neumann conditions on the canonical field, it is important to note that it is the *wave-function renormalised* canonical field on which Dirichlet or Neumann conditions are imposed.

In the case where $\Delta = d/2$ there is no wave-function renormalisation, and as before we can impose either Dirichlet or Neumann conditions on the canonical field.

The wave-functional with Dirichlet conditions is always normalisable, but it is not immediately clear whether this is true for the wave-functional with Neumann conditions. Unitarity constraints on the bulk field reveal that the latter is normalisable if and only if $M_r \leq 1$ [10].

Notice that although this discussion applied to free scalar fields, it applies equally well to interacting bulk scalars (of course there may be additional renormalisations to deal with). Also, it is straightforward to extend the analysis to fields of other spin, and in this way perturbations preserving some supersymmetry could be considered.

4 AdS/CFT Correspondence Formulae

In the usual version of the AdS/CFT correspondence, we equate the partition functions of the conformal boundary theory and the bulk gravitational theory:

$$Z[\phi]_{grav} = Z[\phi]_{CFT} \equiv \langle \exp \int \phi \mathcal{O} \rangle_{CFT}. \tag{15}$$

Here $\hat{\phi}$ on the CFT side is a source for the operator $\mathcal{O}$, which is any scalar primary of the boundary theory. The partition function on the left-hand side can be identified with the wave-functional considered in the last section. On the gravity side it corresponds to a boundary value for the corresponding bulk field. The usual prescription is to diagonalise the boundary value of (2) corresponding to the *larger* scaling dimension, since this always gives a normalisable solution. However, as first found in [2], in the range $d/2 < \Delta < d/2+1$ both asymptotics in (2) are normalisable, and in this case a single bulk field gives rise to two different relevant perturbations of the CFT.

A multi-trace interaction in the boundary theory is obtained by adding an extra term to the boundary action:

$$I_{pert} = I_{CFT} + W[\mathcal{O}], \tag{16}$$

where $W[\mathcal{O}]$ is an arbitrary functional of primary scalar operators. Let us see how this affects the canonical quantisation. For the purposes of illustration it is convenient to consider a double-trace perturbation, so consider the partition function

$$Z_f[\phi] = \langle \exp(-\int \frac{f}{2}\mathcal{O}^2 + \int \mathcal{O}\phi \rangle_{CFT}. \tag{17}$$

The coupling to $\mathcal{O}$ can be linearised with a Hubbard-Stratonovich transformation:

$$Z_f[\phi] = \det{}^{1/2}\left(-\frac{1}{f}\right) \int D\sigma \langle \exp \int \left(\frac{1}{2f}\sigma^2 + (\sigma + \phi)\mathcal{O} \right) \rangle_{CFT}. \tag{18}$$

As we have seen, according to the AdS/CFT correspondence, $Z_0[\phi]$ can be interpreted as the wave-functional of a bulk scalar. This implies that to quadratic order

$$Z_f[\phi] = \det{}^{1/2}\left(-\frac{1}{f}\right) \int D\sigma \exp \int \left(F + \tfrac{1}{2}(\phi+\sigma)\Gamma(\phi+\sigma) + \tfrac{1}{2f}\sigma^2 \right), \tag{19}$$

where F and Γ are the free energy and quadratic kernel given in Section 3. Performing the σ integral, we have

$$Z_f[\phi] = \det{}^{-1/2}\,(f\Gamma + 1)\exp\left(F + \tfrac{1}{2}\phi\tfrac{\Gamma}{1+f\Gamma}\phi\right). \tag{20}$$

Notice that for small f we recover the previous result for $Z_0[\phi]$, while in the limit $f \to \infty$ the kernel Γ is replaced with $1/f$. Now consider how all this is affected by the wave-function renormalisation. Assuming we started with Dirichlet conditions, $\Gamma \sim \tau^{2M_r}$, ϕ is renormalised by $\phi \to \tau^{-M_r}\phi$ as before, and the effect is to send $f \to 0$.

If we started with Neumann conditions on the canonical field then $\Gamma \sim \tau^{-2M_r}$. For any f except $f = 0$ there is no wave-function renormalisation: (20) gives a wave-functional that is finite as the cutoff is removed:

$$Z_f[\phi] = \det{}^{-1/2}\,(f\Gamma + 1)\exp\left(F + \tfrac{1}{2}\phi\tfrac{1}{f}\phi\right). \tag{21}$$

Since the conjugate field is represented on this wave-functional by functional differentiation, we see that this corresponds to imposing the boundary conditions $\alpha = f\beta$ on the bulk field (2), in accordance with the prescription of [8]. Thus the limit $f \to \infty$ corresponds to Neumann conditions on the canonical field (the kernel in (20) naturally tends to zero in this limit because we did not introduce a source for the conjugate field). As pointed out in [7], only for $f = 0$ or ∞ does the bulk propagator respect SO(4,2) invariance. So only in these cases should we expect the AdS/CFT correspondence to work.

In conclusion, we can identify Dirichlet and Neumann conditions on the canonical field with the $f = 0$ and $f = \infty$ limits of (20) respectively. Finite f represents a mixture of Dirichlet and Neumann conditions, but one loop effects will deform the AdS background in this case.

From the gauge theory side the situation is as follows. The perturbation $\frac{1}{2}f\mathcal{O}^2$ drives a renormalisation group flow from a UV fixed point, where $f = 0$, to an IR fixed point, at $f = \infty$. At the fixed points the gravity dual of this theory lives on AdS space, and the UV and IR fixed points correspond resepectively to Dirichlet and Neumann conditions for the bulk field dual to $\mathcal{O}$.

Multi-trace perturbations of dimension more than two are similarly described by the prescription of [8]. If we Fourier transform (21) we get a functional written in terms of a source for the canonical conjugate field

$$Z_f[\pi] = \det{}^{-1/2}\,((f\Gamma + 1)/f)\exp\left(F - \tfrac{1}{2}f\pi^2\right). \tag{22}$$

From this we see that as a result of the wave-function renormalisation the prescription for a perturbation $W[\mathcal{O}]$ is to replace the wave-functional (15) with $W[\pi]$, where π is a boundary value for the *conjugate* of the canonical field. The free energy is also changed as a result of the extra determinant in (22).

5 Central charges and the c-theorem

On general grounds, [16, 17], the Weyl anomaly takes the form $\mathcal{A} = -a\,E - c\,I$ where E is the Euler density, $(R^{ijkl}R_{ijkl} - 4R^{ij}R_{ij} + R^2)/64$, and I is the square of the Weyl tensor, $I = (-R^{ijkl}R_{ijkl} + 2R^{ij}R_{ij} - R^2/3)/64$. The c-theorem in four dimensions [15, 14] suggests that the central charge as defined in [15] (which is related to heat-kernel coefficients and is in general a combination of a and c) should be larger in the ultraviolet than in the infrared. In this section we will check this for a double-trace deformation.

The exact N-dependence of the Weyl anomaly of the boundary CFT was calculated from the AdS/CFT correspondence in [4, 6] (an overview of the complete calculation for the $\mathcal{N} = 4$ SYM/Type IIB gravity correspondence is given in [5]). The leading order result in the large-N expansion was first found by [3], but at subleading order there are contributions from all the Kaluza-Klein modes of supergravity, the contribution of each supergravity field on AdS being given by a universal formula.

In our calculation of the anomaly Dirichlet boundary conditions were assumed for all of the bulk fields (there are no fields with masses in the range for which Neumann conditions are admissible). It would be interesting to consider compactifications (such as Type IIB supergravity on $AdS_5 \times T^{1,1}$) for which there are masses allowing Neumann conditions.

In this section we will extend our result for the Weyl anomaly to theories with double-trace perturbations (or Neumann boundary conditions for some of the bulk fields). As explained in [5], the Weyl anomaly is given by the response of the free energy to a Weyl scaling of the boundary metric, which is equivalent to scaling the cutoff r_0, so that the contribution of a bulk scalar to the anomaly is

$$\int d^d x \sqrt{\hat{g}}\delta\mathcal{A} = \frac{\partial}{\partial r_0} F = \tfrac{1}{2}\mathrm{Tr}\Gamma, \tag{23}$$

where we used the Schrödinger equation (13). The expansion of Γ in powers of the differential operator gives

$$\Gamma = \sum_{n=0}^{\infty} b_n(r_0)\left(\Box + \frac{(d-2)\hat{R}}{4(d-1)}\right)^n , \tag{24}$$

with

$$b_0 = -\sqrt{m^2 + \frac{d^2}{4}} \tag{25}$$

and $b_n \to 0$ as $r_0 \to \infty$ for all $n \neq 0$.

The functional trace is regulated with a Seeley-de Witt expansion of the heat-kernel

$$\mathrm{Tr}\,\Gamma = \sum_{n=0}^{\infty} b_n(r_0)\left(-\frac{\partial}{\partial s}\right)^n \mathrm{Tr}\,\exp\left(-s\left(\Box + \frac{(d-2)\hat{R}}{4(d-1)}\right)\right), \tag{26}$$

$$\mathrm{Tr}\,\exp\left(-s\left(\Box + \frac{(d-2)\hat{R}}{4(d-1)}\right)\right) = \int d^d x\,\sqrt{\hat{g}}\frac{1}{(4\pi s)^{d/2}}\left(a_0 + s\,a_1(x) + s^2\,a_2(x) + s^3\,a_3(x) + ..\right), \tag{27}$$

with s small. Assuming an even-dimensional boundary, in the limit $s \to 0$ and $r_0 \to -\infty$ the only surviving contributions are from $a_0, a_1, \ldots a_{d/2}$. The coefficients of all but $a_{d/2}$ diverge, and can be cancelled by adding counterterms to F, but the finite contribution proportional to $a_{d/2}$ determines the anomaly. Since $\sqrt{m^2 + d^2/4} = \Delta - d/2$, we find that

$$\mathcal{A} = -\frac{\Delta - d/2}{2(4\pi)^{d/2}} a_{d/2}. \tag{28}$$

All this assumed Dirichlet conditions for the canonical bulk field, but we would like to know if Neumann conditions give a different result for the anomaly. To change from Dirichlet boundary conditions to the more general condition $\dot{\varphi} = \lambda \varphi$, we add to (12) the boundary term $\exp(-\frac{\alpha}{2} \int \hat{\varphi}^2)$ and integrate over $\hat{\varphi}$, giving the determinant $\det^{-1/2}(\Gamma - \lambda)$. Since ϕ has the asymptotic form $\alpha \tau^{\Delta - d/2} + \beta \tau^{d/2 - \Delta}$, what we called Neumann boundary conditions (diagonalising β) correspond to $\lambda = d/2 - \Delta$.

We have

$$\det(\Gamma - \lambda) = e^{-2\delta F}, \tag{29}$$

where δF is the correction to the free energy. Thus using (13)

$$\frac{\partial}{\partial r_0} \delta F = \mathrm{Tr} \left(-\frac{\partial_{r_0} \Gamma}{2(\Gamma - \lambda)} \right) = \mathrm{Tr} \left(-\frac{\Gamma^2 - z^2 e^{-\rho} \left(\Box + \frac{(d-2)\hat{R}}{4(d-1)} \right) - M_r^2}{2(\Gamma - \lambda)} \right). \tag{30}$$

As $r_0 \to \infty$ $\Gamma \to -M_r - \frac{1}{2+2M_r} \left(\Box + \frac{(d-2)\hat{R}}{4(d-1)} \right) z^2 + \ldots$ and for $\lambda \neq -M_r = d/2 - \Delta$ (30) tends to zero. For the specific value $\lambda = d/2 - \Delta$, however, it tends to $\mathrm{Tr}(-1)$, giving a correction to the anomaly,

$$\delta \mathcal{A} = -\frac{1}{(4\pi)^{d/2}} a_{d/2}. \tag{31}$$

Notice that for generic mixed boundary conditions there is no correction to the anomaly.

From the point of view of a double-trace perturbation, as we go from the UV (Dirichlet conditions and $f = 0$) to the IR (Neumann conditions and $f = \infty$) (31) implies that the central charge (as defined in [15]) is decreased by 1. This is in accordance with the c-theorem, which predicts that $c_{UV} > c_{IR}$ [15, 14]. The correction to the anomaly applies even in the case $\Delta = d/2$. In other words there is a non-zero flow for bulk fields with masses saturating the Breitenlohner-Freedman bound, in contradiction with the assumption to the contrary made in [7]. This explains the discrepancy with that result. From the gauge theory side the result of [7] was reproduced in [11], but the zeta-function regularisation used there is tantamount to making the same assumption, since it is based on an expansion in odd powers of $\Delta - d/2$.

The result (31) can be extended to all the fields of Supergravity by using the results of [5] in which the Schrödinger equations for bosonic and fermionic fields of higher spin were reduced to the same form as the scalar field equation. This will be discussed in a forthcoming publication.

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